

# NETWORK GROUP DISCOVERY BY HIERARCHICAL LABEL PROPAGATION

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# GROUPS IN NETWORKS

GROUP DETECTION BY PROPAGATION

EMPIRICAL ANALYSIS & COMPARISON

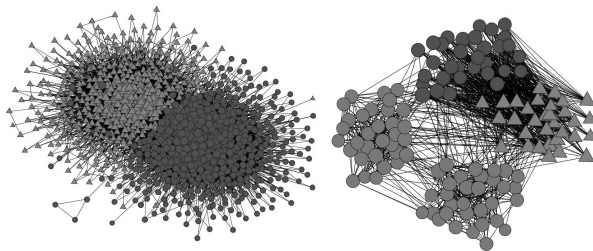
CONCLUSIONS

# NODE GROUPS

**community** densely linked nodes sparsely linked between (Girvan and Newman, 2002)

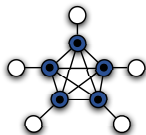
**module** nodes linked to similar other nodes (Newman and Leicht, 2007)

**other** mixtures of these

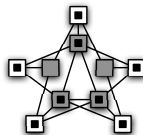


# GROUP FORMALISM

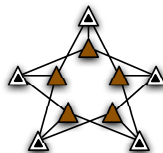
$S$  is group of nodes and  $T$  its linking pattern. (Šubelj et al., 2013)



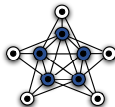
Community ( $S = T$ )



Mixture ( $S \approx T$ )



Module ( $S \neq T$ )



$S$  is shown with filled nodes,  $T$  is shown with marked nodes.

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# LABEL PROPAGATION

Label propagation algorithm: (Raghavan et al., 2007)

$$g_i = \operatorname{argmax}_g \sum_{j \in \Gamma_i} \delta(g_j, g)$$

$g_i$  is group label of node  $i$  and  $\Gamma_i$  are its neighbors.



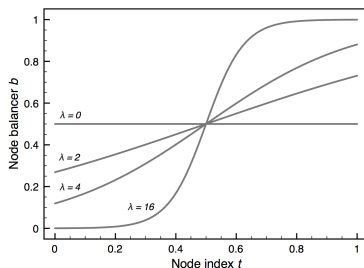
*Algorithm has near linear complexity  $\mathcal{O}(m)$ , where  $m$  is number of links.*

# BALANCED PROPAGATION

Balanced propagation algorithm: (Šubelj and Bajec, 2011a)

$$g_i = \operatorname{argmax}_g \sum_{j \in \Gamma_i} b_j \cdot \delta(g_j, g) \quad b_i = \frac{1}{1 + e^{-\lambda(t_i - \frac{1}{2})}}$$

$b_i$  is balancer of node  $i$  and  $t_i \in (0, 1]$  is its normalized index.



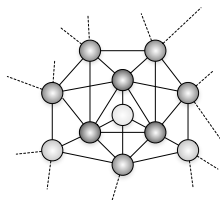
# Partitions found in Zachary network in 1000 runs drops from 184 to 19.

# ADVANCED PROPAGATION

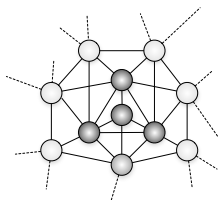
Defensive propagation algorithm: (Šubelj and Bajec, 2011b)

$$g_i = \operatorname{argmax}_g \sum_{j \in \Gamma_i} p_j b_j \cdot \delta(g_j, g)$$

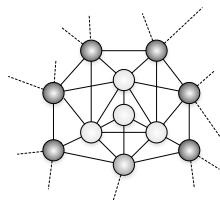
$p_i$  is probability that random walker on group  $g_i$  visits node  $i$ .



By degrees



Defensive



Offensive

*Defensive algorithm has high recall, offensive algorithm has high precision.*

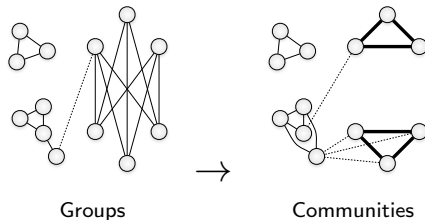


# GENERAL PROPAGATION

General propagation algorithm: (Šubelj and Bajec, 2012)

$$g_i = \operatorname{argmax}_g \left( \overbrace{\tau_g \cdot \sum_{j \in \Gamma_i} p_j b_j \cdot \delta(g_j, g)}^{\text{Community detection}} + (1 - \tau_g) \cdot \overbrace{\sum_{\substack{j \in \Gamma_i \\ k \in \Gamma_j \setminus \Gamma_i}} \frac{p'_j b_k}{k_j} \cdot \delta(g_k, g)}^{\text{Module detection}} \right)$$

$k_i$  is degree of node  $i$  and  $\tau_g \in [0, 1]$  is parameter of group  $g$ .



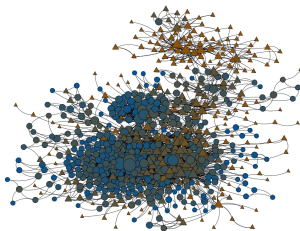
*Group parameters  $\tau$  have to be set accordingly (conductance, clustering).*

# HIERARCHICAL PROPAGATION

Hierarchical propagation algorithm: (Šubelj and Bajec, 2014)

$$\tau_{g_i} = \begin{cases} 1 & \text{if } d_i \geq p \text{ and } \langle d \rangle \geq p \\ 0 & \text{if } d_i < p \text{ and } \langle d \rangle < p \\ 0.5 & \text{else} \end{cases}$$

$d_i$  is corrected clustering of node  $i$  and  $p$  is clustering of configuration model.

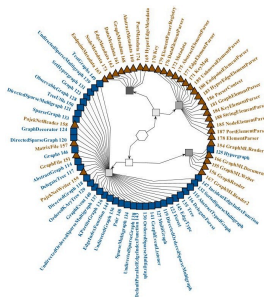


*Communities are in dense parts ( $d \gg 0$ ), modules are in sparse parts ( $d \approx 0$ ).*

# HIERARCHICAL PROPAGATION (II)

Hierarchical propagation algorithm: (Šubelj and Bajec, 2014)

- ▶ group detection by propagation → communities
- ▶ bottom-up group agglomeration → hierarchy
- ▶ top-down group refinement → modules



*Alternative group hierarchies are compared by maximum likelihood.*

GROUPS IN NETWORKS

GROUP DETECTION BY PROPAGATION

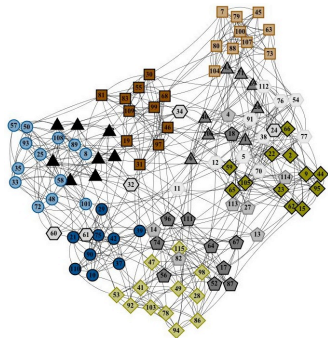
EMPIRICAL ANALYSIS & COMPARISON

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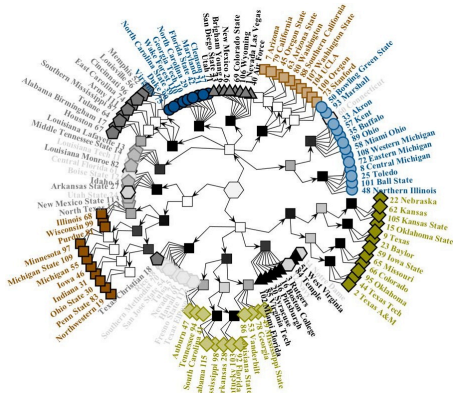
# SOCIAL NETWORKS

Node shapes show sociological division into groups, (Girvan and Newman, 2002)

shades of inner nodes of hierarchy are proportional to link density.



American football network

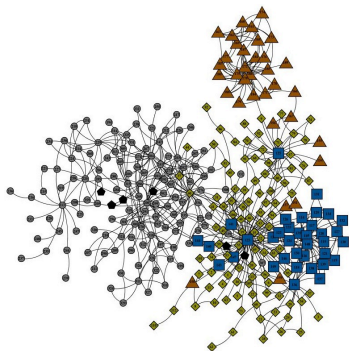


Group hierarchy

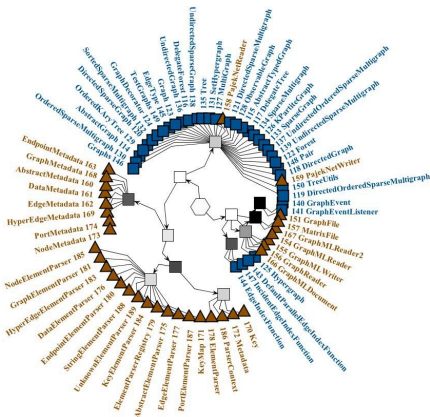
# SOFTWARE NETWORKS

Node shapes show developer division into packages, (O'Madadhain et al., 2005)

shades of inner nodes of hierarchy are proportional to link density.



JUNG dependency network



Group hierarchy

# REAL-WORLD NETWORKS

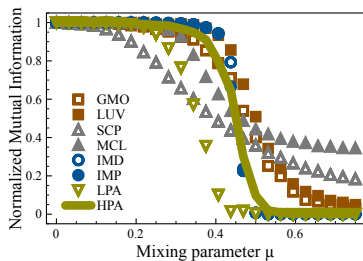
Label propagation algorithm (LPA), multi-stage modularity optimization or Louvain method (LUV), random walk compression or Infomap (IMP),  $k$ -means data clustering (KMN), mixture model with expectation-maximization (EMM) and hierarchical propagation algorithm (HPA).

	Community detection			Group detection		
	LPA	LUV	IMP	KMN	EMM	HPA
American football network	0.892	0.876	<b>0.922</b>	0.845	0.823	0.909
	0.796	0.771	<b>0.890</b>	0.698	0.683	0.850
Southern women network	0.184	0.309	0.417	0.677	0.827	<b>0.932</b>
	0.093	0.174	0.273	0.560	0.720	<b>0.936</b>

Normalized Mutual Information and Adjusted Rand Index

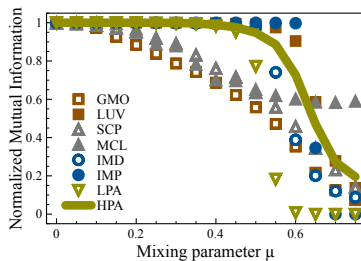
# SYNTHETIC NETWORKS

Greedy optimization of modularity (GMO), multi-stage modularity optimization or Louvain (LUV), sequential clique percolation (SCP), Markov clustering (MCL), structural compression or Infomod (IMD), random walk compression or Infomap (IMP), label propagation algorithm (LPA) and hierarchical propagation algorithm (HPA).



4 communities

(Girvan and Newman, 2002)



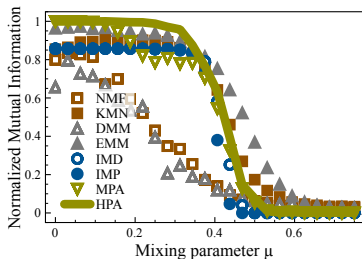
$\geq 10$  communities

(Lancichinetti et al., 2008)



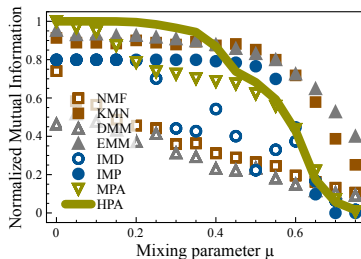
# SYNTHETIC NETWORKS (II)

Symmetric nonnegative matrix factorization (NMF),  $k$ -means data clustering (KMN), (degree-corrected) mixture model (EMM & DMM), structural compression or Infomod (IMD) and random walk compression or Infomap (IMP), model-based propagation algorithm (MPA) and hierarchical propagation algorithm (HPA).



2 communities & bipartite modules

(Šubelj and Bajec, 2012)



3 communities & tripartite modules

(Šubelj and Bajec, 2014)

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Hierarchical propagation algorithm: (Šubelj and Bajec, 2014)

- ▶ non-overlapping community and module detection
- ▶ easy to implement or extend with domain knowledge
- ▶ benefits in group detection, hierarchy discovery, link prediction





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## Group detection in complex networks: An algorithm and comparison of the state of the art



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### HIGHLIGHTS

- We propose a propagation-based algorithm for group detection in complex networks.
- The main novelty is a hierarchical refinement procedure for discovery of different groups.
- The algorithm is comparable to the state of the art and has near ideal complexity.
- We consider group detection, hierarchy discovery and link prediction tasks.

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### ABSTRACT

Complex real-world networks commonly reveal characteristic groups of nodes like communities and modules. These are of value in various applications, especially in the case of large social and information networks. However, while numerous community detection techniques have been presented in the literature, approaches for other groups of nodes are relatively rare and often limited in some way. We present a simple propagation-based algorithm for general group detection that requires no a priori knowledge and has near ideal complexity. The main novelty here is that different types of groups are revealed through an adequate hierarchical group refinement procedure. The proposed algorithm is validated on various synthetic and real-world networks, and rigorously compared against twelve other state-of-the-art approaches on group detection, hierarchy discovery and link prediction tasks. The algorithm is comparable to the state of the art in community detection, while superior in general group detection and link prediction. Based on the comparison, we also discuss some prominent directions for future work on group detection in complex networks.

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